Part I 線性代數（選擇題）

Answer the questions by providing the most appropriate choice. Each correct answer will be given 4 points. Each wrong answer will be given -1 point.

1. (4%) Consider

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & 5 \\ -2 & 1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 8 & -3 & -5 \\ 0 & 1 & 2 \\ 4 & -7 & 6 \end{bmatrix}, \quad \text{and } C = \begin{bmatrix} 0 & -2 & 3 \\ 1 & 7 & 4 \\ 3 & 5 & 9 \end{bmatrix}.$$

Which of the following equalities is invalid ?

(a) $(A-B)^2 = (B-A)^2$
(b) $(AB^{-1})^{-1} = BA^{-1}$
(c) $(AB)^2 = A^2B^2$
(d) $(AB)C = A(BC)$
(e) $A(B+C) = AB + AC$

2. (4%) Let matrix

$$A = \begin{bmatrix} 3 & 5 & -2 & 6 & 0 \\ 1 & 2 & -1 & 1 & 0 \\ 2 & 4 & 1 & 5 & 0 \\ 3 & 7 & 5 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

The determinant of $2A$ is equal to

(a) -18
(b) -576
(c) -1152
(d) -192
(e) -288

3. (4%) Let $\mathbf{u} = (2, -2, 3)$, $\mathbf{v} = (1, -3, 4)$, and $\mathbf{w} = (3, 6, -4)$; also $\|\cdot\|$ denotes the Euclidean norm operator. Which of the following expressions is of the smallest value ?

(a) $\|\mathbf{u}+\mathbf{v}\|$
(b) $\|-2\mathbf{u}\| + 2\|\mathbf{v}\|$
(c) $\|3\mathbf{u} - 5\mathbf{v} + \mathbf{w}\|$
(d) $2\|\mathbf{u}+\mathbf{v}-\mathbf{w}\|$
(e) $\left\|\frac{1}{\|\mathbf{w}\|}\mathbf{w}\right\|$

4. (4%) Let A be an $n \times n$ matrix, and the transformation $T_A : R^n \longrightarrow R^n$ is the multiplication by A. The matrix A is invertible. Which of the following statements is false ?
   (a) The column vectors of A are linearly independent.
   (b) The column vectors of A span R^n.
   (c) The column vectors of A form a basis for R^n.
   (d) A has rank n.
   (e) A has nullity 1.

5. (4%) Which of the following sets of vectors is orthonormal with respective to the Euclidean inner product on R^2?
   (a) $(0,1),(2,0)$
   (b) $(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}),(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$
   (c) $(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}),(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$
   (d) $(0,0),(1,1)$
   (e) $(-1,0),(1,0)$

6. (4%) Let $T : R^4 \mapsto R^3$ be the linear transformation given by the formula

$$T(x_1,x_2,x_3,x_4) = (4x_1 + x_2 - 2x_3 - 3x_4, 2x_1 + x_2 + x_3 - 4x_4, 6x_1 - 9x_3 + 9x_4).$$

   Which of the following vectors is in the kernel of T?
   (a) $(1,2,2,0)$
   (b) $(0,-4,1,0)$
   (c) $(0,0,0,1)$
   (d) $(0,-8,2,0)$
   (e) $(3,-8,2,0)$

7. (4%) Which of the following statements is false?
   (a) If matrices A and B have the same reduced row-echelon form, then they have the same row space.
   (b) If H is a subspace of R^3, then there is a 3×3 matrix A such that H equals the column space of A.
   (c) If A is $m \times n$ and rank$(A) = m$, then the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is one-to-one.
   (d) If A is $m \times n$ and the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is onto, then rank$(A) = m$.
   (e) A change-of-coordinates matrix is always invertible.

8. (4%) Suppose all vectors are in R^n with the standard inner product space. Which of the following statements is false?

(a) The set of all vectors in R^n orthogonal to one fixed vector is a subspace of R^n.

(b) If W is a subspace of R^n, then W and $W^{\perp}$ have no vectors in common.

(c) If $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is an orthogonal set and if c_1, c_2, c_3 are scalars, then $\{c_1\mathbf{v}_1, c_2\mathbf{v}_2, c_3\mathbf{v}_3\}$ is an orthogonal set.

(d) If a square matrix has orthonormal columns, then it also has orthonormal rows.

(e) If W is a subspace, then $\|\text{proj}_W\mathbf{v}\|^2 + \|\mathbf{v} - \text{proj}_W\mathbf{v}\|^2 = \|\mathbf{v}\|^2$.

Part II 線性代數（計算題）

Show all your work. No partial credit will be given without computation details.

1. (10%) Express the matrix

$$A = \begin{bmatrix} 0 & 1 & 7 & 8 \\ 1 & 3 & 3 & 8 \\ -2 & -5 & 1 & -8 \end{bmatrix}$$

in the form $A = EFGR$, where E, F, and G are elementary matrices, and R is in row-echelon form.

2. (8%) Suppose that $\mathbf{u}_1, \mathbf{u}_2$ and $\mathbf{u}_3$ form a linearly independent set, and define

$$\mathbf{v}_1 = \mathbf{u}_1 + \mathbf{u}_2 + \mathbf{u}_3, \quad \mathbf{v}_2 = \mathbf{u}_1 + \alpha\mathbf{u}_2, \quad \text{and} \quad \mathbf{v}_3 = \mathbf{u}_2 + \beta\mathbf{u}_3.$$

Find the conditions on α and β such that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ forms a linearly independent set.

Part III 機率

1. (10%) 大樂透彩券(49 選 6 大樂透)玩法如下: 您必須從 01~49 中任選 6 個號碼進行投注。開獎時，開獎單位將隨機開出六個號碼加一個特別號，這一組號碼就是該期 49 選 6 大樂透的中獎號碼，也稱為「獎號」。您的六個選號中，如果有三個以上（含三個號碼）對中當期開出之六個號碼（特別號只適用於貳獎、肆獎和陸獎），即為中獎，並可依規定兌領獎金。其中彩金較大的前幾項獎項的中獎方式如下表，請問大樂透中(1)參獎(2)肆獎的機率分別是多少? (列出算式即可不需算出數值答案，各配分 5%)

獎項	中獎方式
頭獎	與當期六個獎號完全相同者
貳獎	對中當期獎號之任五碼＋特別號
參獎	對中當期獎號之任五碼
肆獎	對中當期獎號之任四碼＋特別號

2. (10%) Consider the following conditional probability problem. When LAPD (Los Angeles Police Department) caught suspects, LAPD would interrogate (偵訊) these suspects alone or all together. The police will determine one of the above two interrogation strategies randomly and the probabilities for these two strategies are 3/4 (interrogate alone) and 1/4 (interrogate all together), respectively. Statistics show that the probability for a suspect to "Tell the Truth" is different under these two strategies. Here, "Telling the Truth" means (1) the suspect admits his crime only when he did commit the crime (2) the suspect denies when he did NOT commit the crime. Statistics also showed that if the suspect is questioned alone, the probability is 1/6 for a suspect to tell the true. It drops to 1/12 if he is questioned with other partners. Please find the probability for a suspect to tell the truth.

3. (15%) Let $\underline{X}=(X_1,X_2,\cdots,X_n)$ be a length-*n* uniformly distributed binary random sequence. We say $\underline{X}$ contains "*m* bursts" if there are *m* segments of successive ones in $\underline{X}$. For instance, the length-8 sequence (<u>1</u>0<u>11</u>0<u>111</u>) has 3 bursts and the length-7 sequence (0<u>1111</u>0<u>1</u>) has 2 bursts. What is the probability that there are 2 bursts in a length-10 binary random sequence?

4. (15%) Let random variables W=X+2Y, Z=X-2Y where X and Y are two zero mean unit variance Gaussian random variables with correlation coefficient $\rho=-1/2$.
   (1). Find Prob{W=X}
   (2). Find Prob{W>X}
   (3). Find the marginal *pdf.* of W and Z.
   (4). Find the correlation of W and Z.
   (5). Are W and Z uncorrelated? Independent?

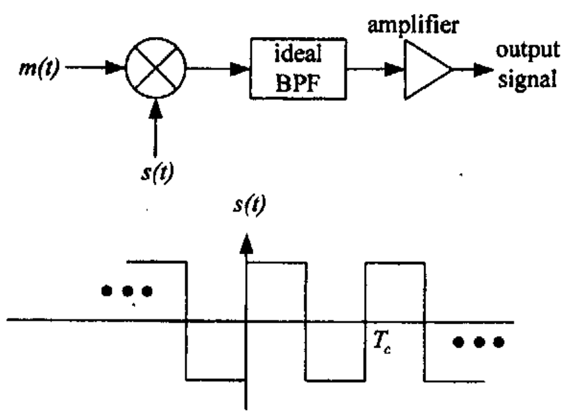
一、選擇題：(共50分) 單選題，每答對一題得五分。

1. An FM signal can be expressed as $u(t)=A_c\cos\left(2\pi f_c t+2\pi k_f\int_{-\infty}^{t} m(\tau)d\tau\right)$, where $m(t)$ is the message signal. Assume that the bandwidth of $m(t)$ is W. Let $\beta=(k_f/W)\max[|m(t)|]$. By Carson's rule, the approximation for the bandwidth of the modulated signal $u(t)$ is
   (A) $(\beta+1)W$
   (B) $(\beta+1)W/2$
   (C) $4(\beta+1)W$
   (D) $2(\beta+1)W$
   (E) None of the above.

2. Suppose that a nonlinear device has an input-output characteristic of the form $v_o(t)=a_1v_i(t)+a_2v_i^2(t)$, where $v_i(t)$ is the input signal and $v_o(t)$ is the output signal, and the parameters a_1 and a_2 are constant. Let W be the bandwidth of $m(t)$. If the input to the nonlinear device is $v_i(t)=m(t)+A_c\cos(2\pi f_c t)$ and an ideal bandpass filter with bandwidth $2W$ centered at $f=f_c$ is employed after the nonlinear device, determine the output signal.
   (A) $A_c m(t)$
   (B) $A_c a_1\left[1+\frac{2a_2}{a_1}m(t)\right]\cos(2\pi f_c t)$
   (C) $(2a_2/a_1)A_c m(t)\cos(2\pi f_c t)$
   (D) $\int_{-\infty}^{\infty}(2a_2/a_1)A_c m(t)\cos(2\pi f_c t)dt$
   (E) None of the above.

3. To reconstruct the original signal $g(t)$ from the sampled signal $g_\delta(t)=\sum_{n=-\infty}^{\infty} g(nT_S)\delta(t-nT_s)$, which of the following statements is correct?
   (A) The original signal may be reconstructed by passing the sampled signal through an envelope detector.
   (B) No matter how fast of the sampling rate is, the original signal cannot be reconstructed from the sampled signal.
   (C) The original signal may be reconstructed by passing the sampled signal through a low pass filter.
   (D) The original signal may be reconstructed by passing the sampled signal through a delta demodulator.
   (E) None of the above.

4. A DSB-SC AM generator is given below. The signal $s(t)$ is a periodic rectangular waveform with period T_c and duty cycle of 50% as shown in the lower part of the figure. Let $f_c = 1/T_c$. Assume that the bandwidth of the message signal $m(t)$ is W with $W << f_c$. Which of the following signal is NOT possible to be generated by this circuit?



(A) $m(t)\sin(4\pi f_c t)$
(B) $m(t)\sin(2\pi f_c t)$
(C) $m(t)\sin(6\pi f_c t)$
(D) $m(t)\sin(10\pi f_c t)$
(E) None of the above.

5. Let $S_X(f)$ be the power-spectral density of a random process $X(t)$. Passing $X(t)$ through a linear system with frequency response $H(f)$ results the output random process $Y(t)$. What is the power of $Y(t)$?

(A) $\int_{-\infty}^{\infty} X(t)H(f)df$

(B) $S_X(f)H(f)$

(C) $\int_{-\infty}^{\infty} S_X(f)|H(f)|^2 df$

(D) $\int_{-\infty}^{\infty} S_X(f)H(f)df$

(E) $\int_{-\infty}^{\infty} X(t)|H(f)|^2 df$

6. Which of the following statements about source coding is false?
(A) The source coding theorem is one of the fundamental theorems of information theory introduced by Shannon in 1948.
(B) Higher entropy the signal has, more bits are required to transmit this signal without loss.
(C) The mutual information between two discrete random variables is related to their conditional entropy.
(D) Huffman coding is lossless.
(E) None of the above.

7. Let $n(t)$ be a white Gaussian noise. Which of the following statements is false?
(A) Knowledge of the mean and autocorrelation of $n(t)$ gives a complete description for $n(t)$.
(B) If $n(t)$ is wide-sense stationary, it is also strict stationary.
(C) For jointly Gaussian process, uncorrelatedness is equivalent to independence.
(D) When $n(t)$ passes through a filter, the output is a Gaussian noise.
(E) None of the above.

8. M-ary simplex waveforms can be constructed by M-ary orthogonal waveforms by $x_m^{'}(t) = x_m(t) - \frac{1}{M}\sum_{n=1}^{M} x_n(t)$, where $x_m(t)$ denotes the M-ary orthogonal waveforms. Which of the following statements is false?
(A) The simplex signal set has smaller energy than the waveforms in the orthogonal set.
(B) The distance between any two points in the simplex signal waveform is the same as that in the orthogonal set.
(C) The simplex signal waveforms are independent of each other.
(D) The crosscorrelation coefficient between any two signal points is a constant dependent on M.
(E) None of the above.

9. Consider a binary PAM signals in which the prior probabilities are $P(s_1) = p$ and $P(s_2) = 1-p$. When these signals are transmitted and corrupted with AWGN, which of the following statements about the optimum MAP detector is false? (Let the received signal and the estimated signal be r and $\hat{x}$, respectively)
(A) $\hat{x} = \arg\max_{x_m}(P(x_m \mid \mathbf{r})$.
(B) $\hat{x} = \arg\max_{x_m}(f(\mathbf{r} \mid x_m)P(x_m))$.
(C) To compute the threshold of binary decision, the power spectrum of the channel noise should be available.
(D) If p=0.5, $\hat{x} = \arg\min_{x_m}\left(r - x_m\right)$.
(E) None of the above.

10. Under the same signal to noise ratio per bit, which of the following modulation signals has the lowest bit error rate by coherent detection?
(A) Binary antipodal signals.
(B) Binary orthogonal signals.
(C) 4-PAM signals
(D) 4-PSK signals.
(E) 4-QAM signals.

二. 計算題 (共 50 分)

1. (15 %) The stationary random process $X(t)$ is passed through an LTI system and the output process is denoted by $Y(t)$. Let the autocorrelation of $X(t)$ be $R_x(\tau)$. Find the output autocorrelation function and the crosscorrelation function between the input and the output processes in each of the following cases.
   (A) (5 %) A delay system with delay Δ.
   (B) (5 %) A system with impulse response $h(t)=\frac{1}{t}$.
   (C) (5 %) A system with $h(t)=e^{-\alpha t}u(t)$ where $\alpha>0$.

2. (10 %) An FM demodulator is shown in Figure 1. The input signal of the demodulator is given by
   $s(t)=A_c\cos\left(2\pi f_c t+2\pi k_f\int_{-\infty}^{t} m(\tau)d\tau\right)$, where $m(t)$ is the message signal.

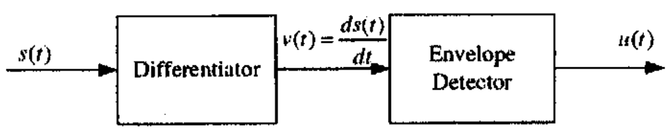


Figure 1.

   (A) (5 %) Determine the signal $v(t)$.
   (B) (5 %) Under what condition the message signal can be recovered using the envelope detector?

3. (10%) Consider the octal signal-point constellation in Figure 2.
   (A) (5%) If the nearest neighbor signal points in 8-QAM constellation are separated by a distance of $2D$ units, determine the radii of the inner and outer circles (a and b).
   (B) (5%) Determine the average transmitter power for the two signal constellation. (Assume that all signal points are equally probable).

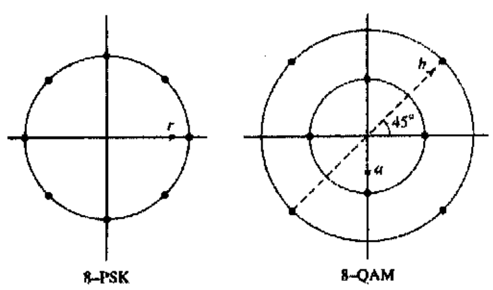


Figure 2.

4. (15%) An input signal $x(t)$ with spectrum bandlimited to W Hz is passed through a lowpass channel which is approximated by $H(f)=\begin{cases}1+\beta\cos 2\pi f t_0, & |\beta|<1,\ |f|\le W\\ 0, & \text{otherwise}\end{cases}$
   (A) (5%) Show that the output $y(t)$ can be expressed as: $y(t)=x(t)+\frac{\beta}{2}[x(t-t_0)+x(t+t_0)]$.
   (B) (5%) If $y(t)$ is passed through a filter matched to $x(t)$, determine the output of the matched filter at $t=kT,\ k=0,\pm1,\pm2,\ldots$, where T is the symbol duration.
   (C) (5%) What is the ISI pattern resulting from the channel when $t_0=T$?

Part I

Answer the questions by providing the most appropriate choice. Each correct answer will be given 4 points. Each wrong answer will be given -1 point.

1. (4%) Consider

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & 5 \\ -2 & 1 & 4 \end{bmatrix}, \; B = \begin{bmatrix} 8 & -3 & -5 \\ 0 & 1 & 2 \\ 4 & -7 & 6 \end{bmatrix}, \text{ and } C = \begin{bmatrix} 0 & -2 & 3 \\ 1 & 7 & 4 \\ 3 & 5 & 9 \end{bmatrix}.$$

Which of the following equalities is invalid ?

(a) $(A-B)^2 = (B-A)^2$
(b) $(AB^{-1})^{-1} = BA^{-1}$
(c) $(AB)^2 = A^2B^2$
(d) $(AB)C = A(BC)$
(e) $A(B+C) = AB + AC$

2. (4%) Let matrix **A** be

$$A = \begin{bmatrix} 3 & 5 & -2 & 6 & 0 \\ 1 & 2 & -1 & 1 & 0 \\ 2 & 4 & 1 & 5 & 0 \\ 3 & 7 & 5 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

The determinant of $2A$ is equal to

(a) -18
(b) -576
(c) -1152
(d) -192
(e) -288

3. (4%) Let A be an $n \times n$ matrix and $\det(A)$ be the determinant of A. Which of the following statements is false ?

(a) If B is the matrix that results when a single row of A is multiplied by a scalar k, then $\det(B) = k\det(A)$.
(b) If B is the matrix that results when two rows of A are interchanged, then $\det(B) = -\det(A)$.
(c) If B is the matrix that results when a multiple of one row of A is added to another row, then $\det(B) = \det(A)$.
(d) If B is the matrix that results when two columns of A are interchanged, then $\det(B) = -\det(A)$.

(e) If A has two proportional columns, then $\det(A) \neq 0$.

4. (4%) Let $\mathbf{u} = (2, -2, 3)$, $\mathbf{v} = (1, -3, 4)$, and $\mathbf{w} = (3, 6, -4)$; also $\|\cdot\|$ denotes the Euclidean norm operator. Which of the following expressions is of the smallest value ?

   (a) $\|\mathbf{u}+\mathbf{v}\|$

   (b) $\|-2\mathbf{u}\| + 2\|\mathbf{v}\|$

   (c) $\|3\mathbf{u} - 5\mathbf{v} + \mathbf{w}\|$

   (d) $2\|\mathbf{u} + \mathbf{v} - \mathbf{w}\|$

   (e) $\left\|\frac{1}{\|\mathbf{w}\|}\mathbf{w}\right\|$

5. (4%) Let $\|\mathbf{u}\|$ be the Euclidean norm of a vector $\mathbf{u}$. Which of the following statements is false ?

   (a) If $\|\mathbf{u}+\mathbf{v}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2$, then $\mathbf{u}$ and $\mathbf{v}$ are orthogonal.

   (b) If $\mathbf{u}$ is orthogonal to $\mathbf{v}$ and $\mathbf{w}$, then $\mathbf{u}$ is orthogonal to $\mathbf{v}+\mathbf{w}$.

   (c) If $\mathbf{u}$ is orthogonal to $\mathbf{v}+\mathbf{w}$, then $\mathbf{u}$ is orthogonal to $\mathbf{v}$ and $\mathbf{w}$.

   (d) If $\|\mathbf{u}-\mathbf{v}\| = 0$, then $\mathbf{u} = \mathbf{v}$.

   (e) If $\|k\mathbf{u}\| = k\|\mathbf{u}\|$ and $\mathbf{u}$ is not a zero vector, then $k \geq 0$.

6. (4%) Which of the following statements is false ?

   (a) If $T : R^n \longrightarrow R^m$ is a one-to-one linear transformation, then there are no distinct vectors $\mathbf{u}$ and $\mathbf{v}$ in R^n such that $T(\mathbf{u}-\mathbf{v}) = \mathbf{0}$.

   (b) If $T : R^n \longrightarrow R^n$ is a linear operator, and if $T(\mathbf{x}) = 2\mathbf{x}$ for some vector $\mathbf{x} \neq \mathbf{0}$, then $\lambda = 2$ is an eigenvalue of T.

   (c) If T maps R^n into R^m, and if $T(c_1\mathbf{u} + c_2\mathbf{v}) = c_1T(\mathbf{u}) + c_2T(\mathbf{v})$ for all scalars c_1 and c_2 and for all vectors $\mathbf{u}$ and $\mathbf{v}$ in R^n, then T is linear.

   (d) If T maps R^n into R^m, and $T(\mathbf{0}) = \mathbf{0}$, then T is linear.

   (e) If $T : R^n \longrightarrow R^m$ is a linear transformation, then $T(\mathbf{0}) = \mathbf{0}$.

7. (4%) The rank of matrix

$$A = \begin{bmatrix} -1 & 2 & 0 & 4 & 5 & 3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}.$$

is equal to

   (a) 0

   (b) 1

   (c) 2

(d) 3

(e) 4

8. (4%) Let A be an $n \times n$ matrix, and the transformation $T_A : R^n \longrightarrow R^n$ is the multiplication by A. The matrix A is invertible. Which of the following statements is false ?

   (a) The column vectors of A are linearly independent.

   (b) The column vectors of A span R^n.

   (c) The column vectors of A form a basis for R^n.

   (d) A has rank n.

   (e) A has nullity 1.

9. (4%) Which of the following sets of vectors is orthonormal with respective to the Euclidean inner product on R^2?

   (a) $(0,1),(2,0)$

   (b) $(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}),(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$

   (c) $(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}),(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$

   (d) $(0,0),(1,1)$

   (e) $(-1,0),(1,0)$

10. (4%) Which of the following expressions is in quadratic form?

   (a) $5x_1^2 - 2x_2^3 + 4x_1x_2$

   (b) $x_1^2 - 7x_2^2 + x_3^2 + 4x_1x_2x_3$

   (c) $x_1^2 - 6x_2^2 + x_1 - 5x_2$

   (d) $(x_1 - 3x_2)^2$

   (e) $x_1^3 - 2x_2^3 + 4x_1$

11. (4%) Let $T : R^4 \mapsto R^3$ be the linear transformation given by the formula

$$T(x_1,x_2,x_3,x_4) = (4x_1 + x_2 - 2x_3 - 3x_4, 2x_1 + x_2 + x_3 - 4x_4, 6x_1 - 9x_3 + 9x_4).$$

   Which of the following vectors is in the kernel of T?

   (a) $(1,2,2,0)$

   (b) $(0,-4,1,0)$

   (c) $(0,0,0,1)$

   (d) $(0,-8,2,0)$

   (e) $(3,-8,2,0)$

12. (4%) Which of the following statements is false?

(a) If matrices A and B have the same reduced row-echelon form, then they have the same row space.
(b) If H is a subspace of R^3, then there is a 3×3 matrix A such that H equals the column space of A.
(c) If A is $m \times n$ and $\text{rank}(A) = m$, then the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is one-to-one.
(d) If A is $m \times n$ and the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is onto, then $\text{rank}(A) = m$.
(e) A change-of-coordinates matrix is always invertible.

13. (4%) Let A and B be square matrices. Which of the following statements is false?
(a) If A is invertible and 1 is an eigenvalue for A, then 1 is also an eigenvalue for A^{-1}.
(b) If A is row equivalent to the identity matrix I, then A is diagonalizable.
(c) If A contains a row or column of zeros, then 0 is an eigenvalue of A.
(d) Each eigenvector of A is also an eigenvector of A^2.
(e) Each eigenvector of an invertible matrix A is also an eigenvector of A^{-1}.

14. (4%) Let A and B be square matrices. Which of the following statements is true?
(a) The eigenvalues of an upper triangular matrix A are exactly the nonzero entries on the diagonal of A.
(b) If a 5×5 matrix A has fewer than 5 distinct eigenvalues, then A is not diagonalizable.
(c) If A is diagonalizable, then the columns of A are linearly independent.
(d) A nonzero vector cannot correspond to two different eigenvalues of A.
(e) A matrix A is invertible if and only if there is a coordinate system in which the transformation $\mathbf{x} \mapsto A\mathbf{x}$ is represented by a diagonal matrix.

15. (4%) Suppose all vectors are in R^n with the standard inner product space. Which of the following statements is false?
(a) The set of all vectors in R^n orthogonal to one fixed vector is a subspace of R^n.
(b) If W is a subspace of R^n, then W and $W^{\perp}$ have no vectors in common.
(c) If $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is an orthogonal set and if c_1, c_2, c_3 are scalars, then $\{c_1\mathbf{v}_1, c_2\mathbf{v}_2, c_3\mathbf{v}_3\}$ is an orthogonal set.
(d) If a square matrix has orthonormal columns, then it also has orthonormal rows.
(e) If W is a subspace, then $\|\text{proj}_W\mathbf{v}\|^2 + \|\mathbf{v} - \text{proj}_W\mathbf{v}\|^2 = \|\mathbf{v}\|^2$.

Part II

Show all your work. No partial credit will be given without computation details.

Please go on to the next page...

1. (10%) Express the matrix

$$A = \begin{bmatrix} 0 & 1 & 7 & 8 \\ 1 & 3 & 3 & 8 \\ -2 & -5 & 1 & -8 \end{bmatrix}$$

in the form $A = EFGR$, where E, F, and G are elementary matrices, and R is in row-echelon form.

2. (10%) Find a basis for the nullspace of

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix}.$$

3. (10%) Find the normalized QR-decomposition of

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 1 & 4 & 6 \end{bmatrix}.$$

4. (10%) Suppose that $\mathbf{u}_1$, $\mathbf{u}_2$ and $\mathbf{u}_3$ form a linearly independent set, and define

$$\mathbf{v}_1 = \mathbf{u}_1 + \mathbf{u}_2 + \mathbf{u}_3, \quad \mathbf{v}_2 = \mathbf{u}_1 + \alpha\mathbf{u}_2, \quad \text{and} \quad \mathbf{v}_3 = \mathbf{u}_2 + \beta\mathbf{u}_3.$$

Find the conditions on α and β such that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ forms a linearly independent set.

End of exam.

1. (15%) The lifetime X of a light bulb is an exponential random variable with parameter λ, i.e., $f_X(x) = \lambda e^{-\lambda x}, x \ge 0$
   a. (5%) Suppose that10 new light bulbs are installed at time $t=0$. Find the probability that all light bulbs are still working at time $t=5$.
   b. (10%) Prove that X does have memoryless property.
2. (10%) Let $G_x(z) = 0.2z^2/(1-0.8z)$ be the probability generating function of discrete random variable X.
   a. Find variance of X.
   b. Find the *pmf* of X.
3. (20%) The random variable **X** and **Y** have the joint probability density function (*pdf.*) $f_{XY}(x,y) = 2e^{-(x+y)}, 0 \le y \le x < \infty$. and the random variable **V, W** satisfy: **V=X+Y, X=W-2Y.**
   a. (5%) Find Prob(X=Y)
   b. (5%) Find Prob(X>Y)
   c. (5%) Find the marginal *pdf.* $f_V(v), f_W(w)$
   d. (5%) Find the **E[VW],COV(V,W)**
4. (10%) Let A and B be given events with $P(A) > 0$, $P(B) \in (0,1)$ and $P(A \mid B)$ all known.
   a. (5%) Determine $P(A \mid B^c)$ in terms of these known quantities. (*ps*: B^c is the complement set of the event B)
   b. (5%) If, in addition, A and B are mutually exclusive, what is the relationship between $P(A)$ and $P(A \mid B^c)$? (Express the relationship in terms of the following four symbols: "≥", "≤", "="or "N/A" for no definite relation)
5. (10%) A fair dice is tossed until 50 ones appear. Find the probability that at least 305 tosses will be necessary.
   a. Derive the exact mathematic formula for this problem. (本題列出算式即可不需算出數值答案)
   b. Express the approximated result in terms of Q-function
6. (10%) A fair coin is tossed 200 times.Find the probability that exactly 100 heads are tossed.
   a. Derive the exact mathematic formula for this problem. (本題列出算式即可

不需算出數值答案)

b. Express the approximated result in terms of Q-function

7. (10%) Let $\underline{X}=(X_1,X_2,\cdots,X_n)$ be a length-*n* uniformly distributed binary random sequence. We say $\underline{X}$ contains "*m* bursts" if there are *m* segments of successive ones in $\underline{X}$. For instance, the length-8 sequence ($\underline{1}0\underline{11}0\underline{111}$) has 3 bursts and the length-7 sequence ($0\underline{1111}0\underline{1}$) has 2 bursts. What is the probability that there are 2 bursts in a length-10 binary random sequence?

8. (15%) Let $X=\cos\Theta$, $Y=\sin\Theta$ where Θ is uniformly distributed in the interval $(0,2\pi)$

a. Find the correlation of *X* and *Y*.

b. Find $f_Y(y\,|\,x)$

c. Find E[*Y*|*X*]

1. Towers of Hanoi: Move *n* disks from peg A to peg B using peg MID as needed. Only one disk at a time may be moved, and this disk must be the top disk on a peg. A larger disk can never be placed on top of a smaller disk.

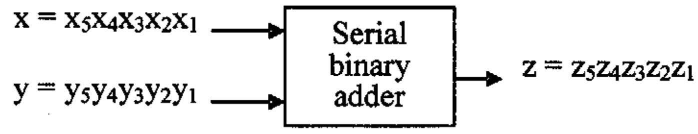
 a. (10 pt.) Find the shortest sequence of moves.

 b. (10 pt.) If direct moves between A and B are disallowed, find the shortest sequence of moves.
 Hint: Each move must be to or from the peg MID.

 c. (10 pt.) Show that, in the process of transferring a tower under the restrictions of the preceding question, we will actually encounter every properly stacked arrangement of n disks on three pegs.
 Hint: There are 3^n possible arrangements.

2. (10 pt.) Let $G = (V, E)$ be an undirected graph or multigraph with no isolated vertices. Prove: G has an Euler circuit if and only G is connected and every vertex in G has even degree.

3. (10 pt.) Build a finite state machine to model a serial binary adder. For example, let $x = x_5x_4x_3x_2x_1 = 00111$ and $y = y_5y_4y_3y_2y_1 = 01101$ be binary numbers where x_1 and y_1 are the least significant bits. We can use such a serial binary adder to obtain $z = x + y$, where $z = z_5z_4z_3z_2z_1$ has the least significant bit z_1. In this example, $z = x + y = 10100$.



4. (15 pt.) Using "Indirect proof" to show "If $5n+4$ is odd, then n is odd".
   Hint: Show contrapositive of the implication.

5. A set is notated with its members and two **braces**. Let
   $\boldsymbol{T} = \{\boldsymbol{m} \mid \boldsymbol{m} \subseteq \{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}\}\}$.
   a. (10 pt.) Show all the members of the set $\boldsymbol{T}$.

   b. (5 pt.) Show all the cardinality of the set $\boldsymbol{T}$.

6. Let R_1 and R_2 be the relations on $\{m, n, p, q\}$ given by
   $$R_1 = \{(m, p), (n, n), (n, p), (q, m)\},$$
   $$R_2 = \{(m, m), (n, n), (p, n), (p, p), (q, n)\}$$
   a. (10 pt.) List the elements of the two composites of R_1 and R_2, including $R_1 \circ R_2$ and $R_2 \circ R_1$.

   b. (10 pt.) Determine whether the relations in (a) are equivalence relations on $\{m, n, p, q\}$ and describe your reasons.
   Hint: A set with the equivalence relation is reflexive, symmetric and transitive.

1. (10 pt.) For a telephone system with m servers, the call blocking probability P can be determined using the following Erlang-B formula.

   Blocking probability $P(m,\rho) = \dfrac{\rho^m / m!}{\sum_{n=0}^{m} \rho^n / n!}$, where m is a positive integer and factor ρ is a constant value. (Note that factorial 0!=1)

   Write a recursive functions that calculates $P(m,\rho)$. (You may write a single or multiple recursive subfunctions to determine P.)

2. The following adjacency matrix represents an undirected Graph G with 6 vertices and 9 arcs. Those values are the arc weights.

	A	*B*	*C*	*D*	*E*	*F*
Vertex *A*	–	7	3	–	–	–
Vertex *B*	7	–	2	6	–	–
Vertex *C*	3	2	–	3	4	–
Vertex *D*	–	6	3	–	2	3
Vertex *E*	–	–	4	2	–	5
Vertex *F*	–	–	–	3	5	–

   (5 pt.) (a) Draw the topology of Graph G.
   (5 pt.) (b) Determine the shortest path tree of Graph G (vertex A is the tree root) and draw the tree.
   (5 pt.) (c) Determine the minimum spanning tree of Graph G.
   (5 pt.) (d) Give the breadth-first traversal of Graph G starting from vertex A.

3.

   (5 pt) (a) Suppose that we have the following key values: 8, 5, 4, 1, 9, 3, 7. Show the heap tree in array implementation after each value is sequentially inserted.

   (10 pt) (b) Use the same data shown in (a) to demonstrate how heap sort works for sorting the above numbers in decreasing order. Be sure

to explain each step in detail.

(5 pt) (c) Analyze the complexity of heap sort in the Big-O notation.

4. Assume that you have some unknown number of records, and three records are shown below. Assume the number of the data does not exceed 1000. Try to use a hash table to store the content. In order to design a hash with associated operations, answer the following questions.

Sample data set with first 3 shown:

ID	Name
D123456789	David
B234567890	Beth
A112233445	Albert
...	...

(3pt.) (a) Use a pseudocode to define a data structure of a hash table and explain your reason for such design.

(7pt.) (b) Design a hash function other than using a direct mapping. Write the algorithm and explain why you believe your function will avoid collision.

(7pt.) (c) Design a collision resolution method. Write the probing algorithm.

(5pt.) (d) Use your hash function to store the above 3 items into your hash table. Draw the resulting table.

(3pt.) (e) Design another data that will cause collision. Use your collision resolution method to resolve the collision and draw the result.

5. Assume you have a text file containing an article (or a report) written in many English sentences. You are asked to keep an account of how many times each word appeared. Using a sentence, "This is a very very beautiful day," as an example. The result will be as the following table. Solve this problem by design a function which uses linked lists.

Word	Count
this	1
is	1
a	1
very	2
beautiful	1
day	1

(5pt.) (a) Use pseudocode to define the data structure of the linked list.

(10pt.) (b) Assume that the entire text is already stored in a linked list. Write the function to parse the text and count the word occurrence. (Notes on the function: State your precondition well. You can have any parameter of your choice. You can assume what to return for your function.)

(10pt.) (C) Analyze your function and comment on its complexity. Give the result in the Big-O notation. Be sure to state details.

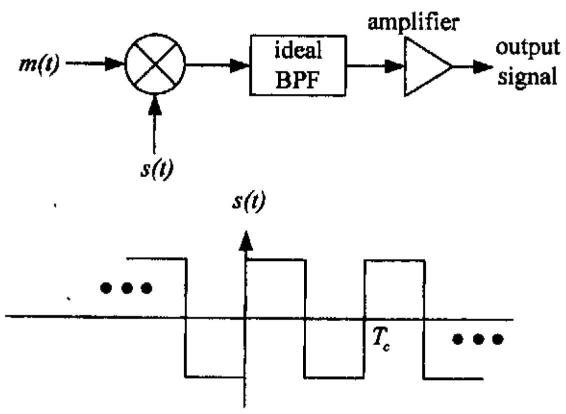
一、選擇題：(共50分) 單選題，每答對一題得五分。

1. An FM signal can be expressed as $u(t)=A_c\cos\left(2\pi f_c t+2\pi k_f\int_{-\infty}^{t}m(\tau)d\tau\right)$, where $m(t)$ is the message signal. Assume that the bandwidth of $m(t)$ is W. Let $\beta=(k_f/W)\max\left[|m(t)|\right]$. By Carson's rule, the approximation for the bandwidth of the modulated signal $u(t)$ is
(A) $(\beta+1)W$
(B) $(\beta+1)W/2$
(C) $4(\beta+1)W$
(D) $2(\beta+1)W$
(E) None of the above.

2. Suppose that a nonlinear device has an input-output characteristic of the form $v_o(t)=a_1v_i(t)+a_2v_i^2(t)$, where $v_i(t)$ is the input signal and $v_o(t)$ is the output signal, and the parameters a_1 and a_2 are constant. Let W be the bandwidth of $m(t)$. If the input to the nonlinear device is $v_i(t)=m(t)+A_c\cos(2\pi f_c t)$ and an ideal bandpass filter with bandwidth $2W$ centered at $f=f_c$ is employed after the nonlinear device, determine the output signal.
(A) $A_c m(t)$
(B) $A_c a_1\left[1+\frac{2a_2}{a_1}m(t)\right]\cos(2\pi f_c t)$
(C) $(2a_2/a_1)A_c m(t)\cos(2\pi f_c t)$
(D) $\int_{-\infty}^{\infty}(2a_2/a_1)A_c m(t)\cos(2\pi f_c t)dt$
(E) None of the above.

3. To reconstruct the original signal $g(t)$ from the sampled signal $g_\delta(t)=\sum_{n=-\infty}^{\infty}g(nT_s)\delta(t-nT_s)$, which of the following statements is correct?
(A) The original signal may be reconstructed by passing the sampled signal through an envelope detector.
(B) No matter how fast of the sampling rate is, the original signal cannot be reconstructed from the sampled signal.
(C) The original signal may be reconstructed by passing the sampled signal through a low pass filter.
(D) The original signal may be reconstructed by passing the sampled signal through a delta demodulator.
(E) None of the above.

4. A DSB-SC AM generator is given below. The signal $s(t)$ is a periodic rectangular waveform with period T_c and duty cycle of 50% as shown in the lower part of the figure. Let $f_c = 1/T_c$. Assume that the bandwidth of the message signal $m(t)$ is W with $W << f_c$. Which of the following signal is NOT possible to be generated by this circuit?



(A) $m(t)\sin(4\pi f_c t)$
(B) $m(t)\sin(2\pi f_c t)$
(C) $m(t)\sin(6\pi f_c t)$
(D) $m(t)\sin(10\pi f_c t)$
(E) None of the above.

5. Let $S_X(f)$ be the power-spectral density of a random process $X(t)$. Passing $X(t)$ through a linear system with frequency response $H(f)$ results the output random process $Y(t)$. What is the power of $Y(t)$?

(A) $\int_{-\infty}^{\infty} X(t)H(f)df$

(B) $S_X(f)H(f)$

(C) $\int_{-\infty}^{\infty} S_X(f)|H(f)|^2 df$

(D) $\int_{-\infty}^{\infty} S_X(f)H(f)df$

(E) $\int_{-\infty}^{\infty} X(t)|H(f)|^2 df$

6. Which of the following statements about source coding is false?
(A) The source coding theorem is one of the fundamental theorems of information theory introduced by Shannon in 1948.
(B) Higher entropy the signal has, more bits are required to transmit this signal without loss.
(C) The mutual information between two discrete random variables is related to their conditional entropy.
(D) Huffman coding is lossless.
(E) None of the above.

7. Let $n(t)$ be a white Gaussian noise. Which of the following statements is false?
(A) Knowledge of the mean and autocorrelation of $n(t)$ gives a complete description for $n(t)$.
(B) If $n(t)$ is wide-sense stationary, it is also strict stationary.
(C) For jointly Gaussian process, uncorrelatedness is equivalent to independence.
(D) When $n(t)$ passes through a filter, the output is a Gaussian noise.
(E) None of the above.

8. M-ary simplex waveforms can be constructed by M-ary orthogonal waveforms by $x_m'(t) = x_m(t) - \frac{1}{M}\sum_{n=1}^{M} x_n(t)$, where $x_m(t)$ denotes the M-ary orthogonal waveforms. Which of the following statements is false?
(A) The simplex signal set has smaller energy than the waveforms in the orthogonal set.
(B) The distance between any two points in the simplex signal waveform is the same as that in the orthogonal set.
(C) The simplex signal waveforms are independent of each other.
(D) The crosscorrelation coefficient between any two signal points is a constant dependent on M.
(E) None of the above.

9. Consider a binary PAM signals in which the prior probabilities are $P(s_1) = p$ and $P(s_2) = 1-p$. When these signals are transmitted and corrupted with AWGN, which of the following statements about the optimum MAP detector is false? (Let the received signal and the estimated signal be r and $\hat{x}$, respectively)
(A) $\hat{x} = \arg\max_{x_m}(P(x_m \mid \mathbf{r})$.
(B) $\hat{x} = \arg\max_{x_m}(f(\mathbf{r} \mid x_m)P(x_m))$.
(C) To compute the threshold of binary decision, the power spectrum of the channel noise should be available.
(D) If p=0.5, $\hat{x} = \arg\min_{x_m}(r - x_m)$.
(E) None of the above.

10. Under the same signal to noise ratio per bit, which of the following modulation signals has the lowest bit error rate by coherent detection?
(A) Binary antipodal signals.
(B) Binary orthogonal signals.
(C) 4-PAM signals
(D) 4-PSK signals.
(E) 4-QAM signals.

二. 計算題 (共 50 分)

1. (15 %) The stationary random process $X(t)$ is passed through an LTI system and the output process is denoted by $Y(t)$. Let the autocorrelation of $X(t)$ be $R_x(\tau)$. Find the output autocorrelation function and the crosscorrelation function between the input and the output processes in each of the following cases.
   (A) (5 %) A delay system with delay Δ.
   (B) (5 %) A system with impulse response $h(t)=\frac{1}{t}$.
   (C) (5 %) A system with $h(t)=e^{-\alpha t}u(t)$ where $\alpha>0$.

2. (10 %) An FM demodulator is shown in Figure 1. The input signal of the demodulator is given by
   $s(t)=A_c\cos\left(2\pi f_c t+2\pi k_f\int_{-\infty}^{t} m(\tau)d\tau\right)$, where $m(t)$ is the message signal.

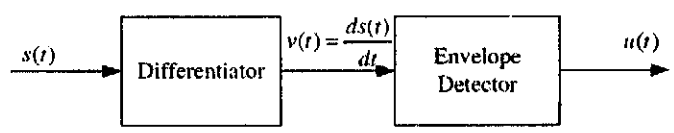


Figure 1.

   (A) (5 %) Determine the signal $v(t)$.
   (B) (5 %) Under what condition the message signal can be recovered using the envelope detector?

3. (10%) Consider the octal signal-point constellation in Figure 2.
   (A) (5%) If the nearest neighbor signal points in 8-QAM constellation are separated by a distance of $2D$ units, determine the radii of the inner and outer circles (a and b).
   (B) (5%) Determine the average transmitter power for the two signal constellation. (Assume that all signal points are equally probable).

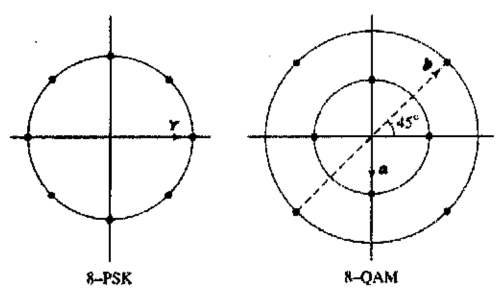
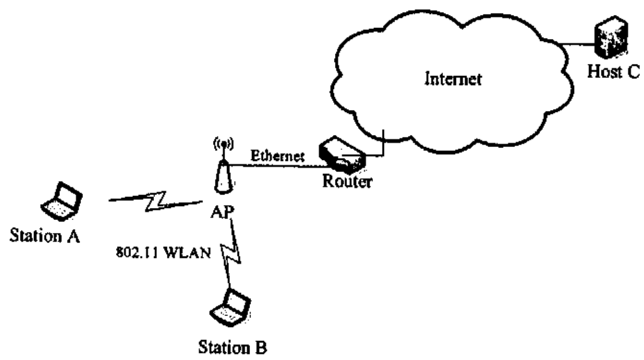


Figure 2.

4. (15%) An input signal $x(t)$ with spectrum bandlimited to W Hz is passed through a lowpass channel which is approximated by $H(f)=\begin{cases}1+\beta\cos 2\pi f t_0, & |\beta|<1,\ |f|\le W\\ 0, & \text{otherwise}\end{cases}$
   (A) (5%) Show that the output $y(t)$ can be expressed as: $y(t)=x(t)+\frac{\beta}{2}\left[x(t-t_0)+x(t+t_0)\right]$.
   (B) (5%) If $y(t)$ is passed through a filter matched to $x(t)$, determine the output of the matched filter at $t=kT,\ k=0,\pm1,\pm2,\ldots$, where T is the symbol duration.
   (C) (5%) What is the ISI pattern resulting from the channel when $t_0=T$?

1. (18 points) Suppose that there are N active nodes in an ALOHA network. Each node transmits with probability p.
   (a) Find the efficiency of the ALOHA network in terms of N and p.
   (b) Find the value of p that maximizes the efficiency.
   (c) Using the value of p found above, find the efficiency of ALOHA by letting N approach infinity

2. (12 points) In 802.11 Wireless LAN MAC frames, there are four address fields. The four address fields are used in conjunction with the "To AP/DS" and "From AP/DS" bits in the frame control field.
   (a) When a frame is sent from a host C remotely in the Internet to station A via AP, what are the four addresses and the "To AP/DS" and "From AP/DS" bits in the frame observed in the wireless LAN?
   (b) When a frame is sent from station A to another station B in the same BSS via AP, what are the four addresses and the "To AP/DS" and "From AP/DS" bits in the frame transmitted by Station A?



	IP Address	MAC address
Station A	140.123.115.100	00:21:91:aa:aa:aa
Station B	140.123.115.102	00:21:91:bb:bb:bb
Host C	140.123.107.101	00:19:5b:cc:cc:cc
AP (wireless interface)		00:21:91:ab:cd:ef
AP (Ethernet interface)		00:03:3c:ab:cd:ef

3. (8 points) Stop-and-Wait, Go-back-N, and Selective Repeat are three ARQ protocols.
   (a) What is the benefit of Go-back-N over Stop-and-Wait?
   (b) What is the possible benefit of Selective Repeat over Go-back-N?

4. (12 points) Routers and bridges.
   (a) Do routers have IP addresses? If so, how many?
   (b) Do bridges have IP addresses? If so, how many?
   (c) What is the 32-bit binary equivalent of the IP address 223.27.124.27?

5. (15 points) Draw a picture with 200 English words to illustrate the differences between Client-Sever architecture and P2P architecture.

6. (15 points) When we deal with firewall configurations, we need to decide which FTP service (either active or passive FTP) should be used. To clarify this common question, please define/compare the active and passive FTP services respectively.

7. (10 points) TCP connection is able to guarantee reliable data communications and provide flow and congestion controls. However, UDP can be very useful in certain situations. List 5 key advantages of using UDP.

8. (10 points) The Internet Assigned Numbers Authority (IANA) maintains the official assignments of TCP/UDP port numbers for specific applications. Fill out the application/protocol names for the well known ports at the following table:

Port Number	Application/Protocol Name
80	Hypertext Transfer Protocol (HTTP)
21	
22	
23	
25	
443	