國立中正大學

108 學年度碩士班招生考試

試 題

[第 2 節]

系所組別	電機工程學系－信號與媒體通訊組
	通訊工程學系－通訊甲組 通訊丙組
科目名稱	線性代數

－作答注意事項－

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

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3. 入場後於考試開始 40 分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

1. **A**, **B**, and **C** are matrices in $\mathbf{R}_{4\times 4}$.

$$\mathbf{A} = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & a \\ 0 & 0 & 4 & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ b & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \mathbf{C} = \mathbf{A} \times \mathbf{B}.$$

a. (5 pts.) Find all possible a and b to satisfy that the inverse matrix of **C** does not exist.

b. (5 pts.) Find all possible b to satisfy that **B** has full column rank.

c. (5 pts.) Find the null space of **B** if $b = 0$.

d. (10 pts.) Show the LU-decomposition of **A** with details.

2. (10 pts.) An overdetermined system

$$x_1 + x_3 = 1$$
$$x_2 + x_3 = 0$$
$$2x_1 + x_3 = 2$$
$$x_1 + cx_3 = d$$
$$x_1 + x_2 = e$$

is consistent. Find all possible values of c, d and e in **R**.

3. Let $\mathbf{D} = \begin{bmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$.

a. (5 pts.) In row space of **D**, find a basis B from row vectors of **D**.

b. (15 pts.) Find eigenvalues and eigenvectors of $\boldsymbol{D}$.

c. (15 pts.) Find a matrix $\boldsymbol{P}$ that diagonalizes **D**.

4. In $\mathbf{P}_2$, define a linear operation

$\boldsymbol{T}(p + qx + rx^2) = (-p + 3q + 2r) + (2p + 2r)x + (4p + q + 5r)x^2$.

a. (15 pts.) Find a basis for the kernel of $\boldsymbol{T}$.

b. (15 pts.) Find a basis for the range of $\boldsymbol{T}$.

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試題

[第3節]

系所組別	通訊工程學系-通訊乙組
科目名稱	線性代數與計算機組織

—作答注意事項—

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Linear Algebra

$$A = \begin{bmatrix} 2 & -1 & 0 & 1 \\ 0 & 3 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & 3 \end{bmatrix},\ \boldsymbol{b} = \begin{bmatrix} 4 \\ 8 \\ 12 \\ 16 \end{bmatrix}$$

Use A and $\boldsymbol{b}$ to solve the questions below:

1.(10 pts.) Evaluate the determinant of A by cofactor expansion.

2.(10 pts.) Use the column vectors of A to finish the linear combination of $\boldsymbol{b}$.

3.(5 pts.) Show the nullity of A.

4.(20 pts.) Find the eigenvalues and eigenvectors of A.

5.(5 pts.) Is A diagonalizable? Show your answer with details.

Computer Organization

6. Improving only the execution time of part of the routings may enhance performance of program execution in multiprocessors systems. The following table shows the execution time of five consecutive routines of a program running on different numbers of processors.

	# Processors	Routine A	Routine B	Routine C	Routine D	Routine E
Case A	2	20 ms	80 ms	10 ms	70 ms	5 ms
Case B	16	4 ms	14 ms	2 ms	12 ms	2 ms

(a) (5 pts.) Find the total execution time of two cases, with 2 and 16 processors, respectively.

(b) (6 pts.) By how much is the total time reduced if the time of routines A, C, and E is improved by 15%, in these two cases?

(c) (6 pts.) By how much is the total time reduced if only routine B is improved by 10%, in two cases?

7. (16 pts.) Give real examples of instruction set design and explain them for each of four underlying principles of hardware design.

Simplicity favors regularity

Smaller is faster

Good design demands compromise

Make the common case fast

8. For a direct-mapped cache design with 32-bit address, the following bits of the address are used to access the cache.

	Tag bits	Index bits	Offset bits
Case A	31 - 10	9 - 4	3 - 0
Case B	31 - 12	11 - 5	4 - 0

(a) (5 pts.) What is the cache line size (in words) in both cases? How many entries does the cache have in both cases? (A word has four bytes.)

(b) (6 pts.) What is the ratio between total bits required for such a cache implementation over the data storage bits in both cases? (A cache implementation includes a valid bit, tag field, and data storage.)

(c) (6 pts.) Starting from power on, the following byte-addressed cache references are recorded.

0 (0x0000),	4 (0x0004),	16 (0x0010),	132 (0x0084),
232 (0x00e8),	160 (0x00a0),	1024 (0x0400),	30 (0x001e),
140 (0x008c),	3100 (0x0c1c),	180 (0x00b4),	2180 (0x0884).

What is the hit ratio in both cases?

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試題

[第1節]

系所組別	電機工程學系－信號與媒體通訊組
	通訊工程學系－通訊甲組
科目名稱	通訊原理

－作答注意事項－

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I. Short Questions

Answer the questions below by providing the most appropriate choice. Write down the correct answer on your answer sheet. No explanations will be considered in grading this portion of the exam. Each correct answer is worth 5 points (5%).

1. To combat the effects of aliasing in practice, which of the following methods is correct?
   (a) Use a high-pass filter to attenuate the in-band noise.
   (b) The signal is sampled at a rate slightly lower than the Nyquist rate.
   (c) Reduce the signal amplitude so that aliasing effect can be reduced.
   (d) Use a low-pass filter to attenuate those high frequency components of the signal before sampling.
   (e) None of the above.

2. A PAM system produces the signal $s(t) = \sum_{n=-\infty}^{\infty} m(nT_s)h(t - nT_s)$, where T_s is the sampling period and $m(t)$ is the message signal. Let $H(f)$ be the Fourier transform of $h(t)$ and $M(f)$ be the Fourier transform of $m(t)$. Determine the Fourier transform of $s(t)$.
   (a) $S(f) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} H\left(f - \frac{k}{T_s}\right) M(f)$
   (b) $S(f) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} M\left(f - \frac{k}{T_s}\right) \otimes H(f)$, where $\otimes$ denotes the convolution operator.
   (c) $S(f) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} M\left(f - \frac{k}{T_s}\right) H(f)$
   (d) $S(f) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} M\left(f - \frac{k}{T_s}\right) H(f)\exp(j2\pi kfT_s)$
   (e) None of the above.

3. Let $m(t)$ be the message signal and W be the bandwidth of $m(t)$. The double-sideband suppressed carrier (DSB-SC) amplitude modulation has the modulated signal $u(t) = A_c m(t)\cos(2\pi f_c t)$. Let the received signal be $r(t) = u(t)$. Suppose we demodulate the received signal by first multiplying $r(t)$ by a locally generated signal $\cos(2\pi f_c t + \phi)$, and then passing the product signal through an ideal low pass filter with bandwidth W. What will happen when $\phi = \pi/2$?
   (a) The demodulated signal is zero.
   (b) The demodulated signal has the maximum signal strength.
   (c) The demodulated signal has the best signal-to-noise ratio.
   (d) The demodulated signal has large frequency components.
   (e) None of the above.

4. For both FM and PM with sinusoidal message signal, we have $u(t) = A_c\cos(2\pi f_c t + \beta\sin(2\pi f_m t))$. Let $J_n(\beta)$ be the Bessel function of the first kind of order n. What is the Fourier transform of $u(t)$?
   (a) $\frac{A_c}{2}\sum_{n=-\infty}^{\infty} J_n(\beta)\left(\delta(f - nf_m) + \delta(f + nf_m)\right)$
   (b) $\sum_{n=-\infty}^{\infty} A_c J_n(\beta)\cos(2\pi(f_c + nf_m))$
   (c) $\frac{A_c}{2}\sum_{n=-\infty}^{\infty} J_n(\beta)\left(\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m)\right)$
   (d) $\sum_{n=-\infty}^{\infty} A_c J_n(\beta)\sin(2\pi(f_c + nf_m))$
   (e) None of the above.

5. Determine the power spectral density of $m(t) = a\cos(2\pi f_m t)$.

   (a) $\frac{a^2}{4}\delta(f-f_m) + \frac{a^2}{4}\delta(f+f_m)$

   (b) $\frac{a^2}{2}\delta(f-f_m) + \frac{a^2}{2}\delta(f+f_m)$

   (c) $\frac{a}{2}\delta(f-f_m) + \frac{a}{2}\delta(f+f_m)$

   (d) $a^2\delta(f-f_m) + a^2\delta(f+f_m)$

   (e) None of the above.

6. A digital communication system generates the transmitted signal $u(t) = \sum_{n=-\infty}^{\infty} a_n h(t - nT_b)$, where a_n is an i.i.d. random process with $P\{a_n = +1\} = P\{a_n = -1\} = 1/2$, and T_b is the symbol period. Define $H(f)$ as the Fourier transform of $h(t)$. Determine the power spectral density of $u(t)$.

   (a) $\frac{1}{T_b}H(f)$

   (b) $\frac{1}{T_b}|H(f)|^2$

   (c) $T_b|H(f)|^2$

   (d) $T_b H(f)$

   (e) None of the above.

7. Let $X(f)$ be the Fourier transform of $x(t)$. Which of the following is a necessary and sufficient condition for $x(t)$ to satisfy

$$x(nT) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

   (a) $\sum_{n=0}^{\infty} X\left(f + \frac{m}{T}\right) = T$

   (b) $\sum_{n=-\infty}^{\infty} X\left(f + mT\right) = T$

   (c) $\sum_{n=0}^{\infty} X\left(f + mT\right) = T$

   (d) $\sum_{n=-\infty}^{\infty} X\left(f + \frac{m}{T}\right) = T$

   (e) None of the above.

8. A binary communication system has the received signal

$$r(t) = s(t) + n(t) \quad \text{for } 0 \le t \le T$$

where $s(t)$ is the transmitted signal with $P\{s(t) = s_A(t)\} = P\{s(t) = s_B(t)\} = 1/2$, $n(t)$ is the AWGN with power spectral density $N_0/2$, and T is the symbol period. Assume that the ML receiver is employed. Determine the bit error rate.

   (a) $Q\left(\sqrt{\frac{d}{2N_0}}\right)$, where $d = \int_0^T |s_A(t) - s_B(t)|^2 dt$

   (b) $Q\left(\sqrt{\frac{d}{2N_0}}\right)$, where $d = \int_0^T |s_A(t) - s_B(t)| dt$

   (c) $Q\left(\sqrt{\frac{d}{N_0}}\right)$, where $d = \int_0^T |s_A(t) - s_B(t)|^2 dt$

   (d) $Q\left(\sqrt{\frac{d^2}{2N_0}}\right)$, where $d = \int_0^T |s_A(t) - s_B(t)|^2 dt$

   (e) None of the above.

9. Two signal vectors are employed for a binary communication system which are given by

$$\mathbf{s}_1 = (+A, +A)$$
$$\mathbf{s}_2 = (+A, -A)$$

The received signal can be represented as

$$\mathbf{r} = \mathbf{s} + \mathbf{w}$$

where $P\{\mathbf{s} = \mathbf{s}_1\} = P\{\mathbf{s} = \mathbf{s}_2\} = 1/2$ and $\mathbf{w} = [w_1, w_2]$ is a 2-dimensinal zero-mean Gaussian noise vector with $E[|w_i|^2] = N_0/2$ for $i = 1, 2$ and $E[w_1 w_2] = 0$. What is the bit error rate for the optimum detector?

(a) $Q\left(\frac{A^2}{N_0}\right)$

(b) $Q\left(\sqrt{\frac{2A^2}{N_0}}\right)$

(c) $Q\left(\sqrt{\frac{A^2}{2N_0}}\right)$

(d) $Q\left(\sqrt{\frac{A^2}{N_0}}\right)$

(e) None of the above.

10. Under the same signal-to-noise ratio, which of the following binary modulation techniques has the best performance.

(a) Binary pulse position modulation.

(b) Binary frequency-shift keying.

(c) On-off keying.

(d) Binary phase-shift keying.

(e) None of the above.

II. Long Questions
Give detailed derivations on the following questions. The grade of this portion depends not only on the correct answers but also on the explanations and derivations. Therefore, explain every detail as possible as you can.

1. (25%) Consider the case of binary PAM signal over the AWGN channel with received signal
$$r = s + w.$$
where s is the transmitted signal and w is the AWGN with variance $N_0/2$. The two possible signal points are $s_1 = -s_2 = \sqrt{E_b}$, where E_b is the energy per bit. The prior probabilities are $P[s = s_1] = p$ and $P[s = s_2] = 1 - p$.
   (a) (10 %) For the optimum detector, a threshold λ can be used to make decision on the transmitted signal. If $r > \lambda$, the decision $s = s_1$ is made and if $r < \lambda$, the decision $s = s_2$ is made. Determine the threshold λ.
   (b) (15 %) Given the optimum detector, what is the bit error probability?

2. (25%) The discrete sequence
$$r_k = \sqrt{E_c}c_k + n_k, \quad \text{for } k = 1, 2, \ldots, n$$
represents the output sequence of samples from a demodulator, where $c_k = \pm 1$ are elements of one of two possible codewords $\mathbf{c}_1 = [1, 1, \ldots, 1]$ and $\mathbf{c}_2 = [1, 1, \ldots, 1, -1, -1, \ldots, -1]$. The codeword $\mathbf{c}_1$ is an all-one vector and $\mathbf{c}_2$ has w elements which are $+1$ and $n - w$ elements which are -1, where w is some positive integer. The noise sequence $\{n_k\}$ is white Gaussian with variance σ^2.
   (a) (10%) What is the maximum-lilelihood detector for the two possible transmitted signals?
   (b) (10%) Determine the error probability as a function of the parameter (σ^2, E_b, w).
   (c) (5%) What is the value of w that minimizes the error probability?

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試 題

[第 3 節]

系所組別	通訊工程學系－通訊乙組
科目名稱	線性代數與資料結構

—作答注意事項—

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國立中正大學 108 學年度碩士班招生考試試題

科目名稱：線性代數與資料結構

系所組別：通訊工程學系-通訊乙組

Linear Algebra

$$A = \begin{bmatrix} 2 & -1 & 0 & 1 \\ 0 & 3 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & 3 \end{bmatrix},\ b = \begin{bmatrix} 4 \\ 8 \\ 12 \\ 16 \end{bmatrix}$$

Use ***A*** and ***b*** to solve the questions below:

1. (10 pts.) Evaluate the determinant of ***A*** by cofactor expansion.
2. (10 pts.) Use the column vectors of ***A*** to finish the linear combination of ***b***.
3. (5 pts.) Show the nullity of ***A***.
4. (20 pts.) Find the eigenvalues and eigenvectors of ***A***.
5. (5 pts.) Is ***A*** diagonalizable? Show your answer with details.

Data Structure

6. (10 pts.) Using C or pseudocode to write a recursive program that reads a string of characters from the keyboard and prints them reversed.

7. Consider sorting techniques and answer the following questions.
   (a) (15 pts.) Using C or pseudocode to write a function to perform sorting for a given integer array.
   (b) (5 pts.) Analyze the time complexity of your algorithm. Be sure to show all detail calculation.

8. Answer the following questions related to graph.
   (a) (5 pts.) Define an adjacency matrix using C or pseudocode. Show the content of your matrix using the graph in Fig. 1.

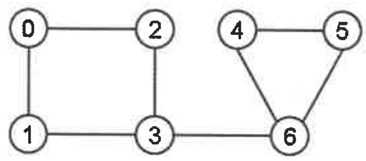

Fig. 1

   (b) (15 pts.) Using C or pseudocode to write an algorithm that returns true if the graph is connected and false if it is disjoint.

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試 題

[第2節]

系所組別	通訊工程學系－通訊甲組
科目名稱	機率

－作答注意事項－

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1. (30%) Let X and Y be two random variables with the following joint probability density function (pdf):

$$f_{X,Y}(x,y) = \begin{cases} 6e^{-x}e^{-2y} & , \text{ if } 0 < x < y < \infty \\ 0 & , \text{ otherwise.} \end{cases} \quad (1)$$

   (a) (10%) Find the marginal pdf of X.

   (b) (10%) Find the expected value of Y.

   (c) (10%) Are X and Y independent or not? Please explain your answer. (0 point if there is no explanation or if the explanation is wrong.)

2. (10%) Suppose that a point (x, y) is chosen uniformly from the circular region $\mathcal{D} = \{(x,y)|\ x^2+y^2 \le 1\}$. Define $\mathcal{S} = \{(x,y)| -1/\sqrt{2} \le x \le 1/\sqrt{2},\ -1/\sqrt{2} \le y \le 1/\sqrt{2}\}$ What is the conditional probability that $x + y \ge 1$ given the fact that (x, y) is in $\mathcal{S}$?

3. (10%) You are given two coins, one standard fair coin and one coin with heads on both sides. You randomly select a coin and flip it once, obtaining a head. Whats the probability that you have selected the fair coin?

4. (10%) Let X be a random variable with the following probability mass function (PMF):

$$p_X(k) = \begin{cases} 1/6 & , k = -1 \\ 1/2 & , k = 1 \\ 1/3 & , k = 2 \end{cases} \quad (2)$$

   Let $Y = e^{X^2}$. Find the cumulative distribution function (CDF) of Y.

5. (10%) Suppose that X is a continuous random variable with cumulative distribution function (CDF) $F_X(x)$ and probability density function (pdf) $f_X(x)$, respectively. Let $Y = |2X|$. Find the CDF of Y.

6. (10%) Let X and Y be two continuous random variables with joint probability density function (pdf) $f_{X,Y}(x,y)$. Let $U = aX + b$ and $V = cY + d$, where a, b, c, d are positive constants. Find the joint probability density function (pdf) of U and V.

7. (20%) Let X be the number of independent Bernoulli trials one needs to carry out until the occurrence of the first success. Let p be the probability of success in each Bernoulli trial.

   (a) (10%) Find the moment generating function of X.

   (b) (10%) Find the expected value of X using the moment generating function obtained in part (a).

國立中正大學
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試 題

[第 1 節]

系所組別	通訊工程學系- 通訊丙組
科目名稱	機率

—作答注意事項—

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1. (10%) Let X be a Gaussian random variable with mean value 0 and variance 1. Let Y be defined as $Y = |X|$. Find the expected value of Y.

2. (10%) A bag contains a number of coins, one of which is a two-headed coin and the rest are fair coins. A coin is selected at random and tossed. If the probability that the toss results in a head is 5/9, then how many fair coins are in the bag?

3. (10%) Ten passengers get on an airport shuttle at the airport. The shuttle has a route that includes 6 hotels, and each passenger gets off the shuttle at his/her hotel. The driver records how many passengers leave the shuttle at each hotel. How many different records can exist?

4. (20%) Let X be a random variable with the following cumulative distribution function (CDF)

$$F_X(x) = \begin{cases} 0 & , x < -1 \\ x/3 + 1/3 & , -1 \le x < 0 \\ x/6 + 2/3 & , 0 \le x < 2 \\ 1 & , 2 \le x \end{cases}$$

   (a) (10%) Find $P(-1 < X < 0)$.

   (b) (10%) Find the expected value of X.

5. (10%) Suppose X is a continuous uniform random variable on $(0, 1)$. Compute the probability density function (pdf) of Y, where $Y = \sin(\pi X)$.

6. (30%) Let the joint cumulative distribution function (CDF) of two random variables X and Y be given as

$$F_{X,Y}(x, y) = \begin{cases} \frac{y+e^{-x(y+1)}}{y+1} - e^{-x} & , \text{if } x > 0 \text{ and } y > 0, \\ 0 & , \text{otherwise.} \end{cases} \quad (1)$$

   (a) (10%) Find the joint probability density function (pdf) of X and Y.

   (b) (10%) Are X and Y independent or not? Please explain your answer. (0 point if there is no explanation or if the explanation is wrong.)

   (c) (10%) Find the probability of the event $\{1 < Y < 2\}$.

7. (10%) Let $X_1, X_2, \ldots, X_{100}$ be independent and identically distributed random variables with zero mean and variance 1/2. Use the central limit theorem and the Q-function to provide an estimate for

$$P\left(\sum_{k=1}^{100} X_k < 25\right). \quad (2)$$