

國立中正大學103學年度學士班二年級轉學生招生考試試題

數學系、地球與環境科學系、物理學系

學系別：資訊工程學系、電機工程學系、機械工程學系

科目：微積分

化學工程學系、通訊工程學系

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一、單選題 (每題4分, 共16分)

1. A rectangle has its base on the x -axis and its upper two vertices on the parabola $y = 6 - x^2$. What is the largest area the rectangle can have?

- (A) $4\sqrt{2}$ (B) $6\sqrt{2}$ (C) 10 (D) 12 (E) None of the above

2. Find the area of the region enclosed by the curve $y = x \sin x$ and the x -axis on the interval $4\pi \leq x \leq 5\pi$.

- (A) 8π (B) 9π (C) 4π (D) 5π (E) None of the above

3. Find the limit: $\lim_{x \rightarrow 0^+} (e^x + x)^{1/x}$

- (A) 1 (B) e (C) e^2 (D) e^4 (E) None of the above

4. If $f(5) = 4$ and $f'(5) = 2/3$, find $(f^{-1})'(4)$

- (A) $3/2$ (B) $2/3$ (C) $1/4$ (D) $4/5$ (E) None of the above

二、複選題(每題6分, 共24分) 請注意: 每題有一個或者一個以上正確答案, 答案完全正確得6分, 否則得0分.

1. Which of the following must be true?

(A) $\lim_{n \rightarrow \infty} \tan\left(\frac{1}{n}\right) = 0$ (B) $\lim_{n \rightarrow \infty} \sqrt[3]{3^n + 2^n} = 3$

(C) $\lim_{n \rightarrow \infty} \frac{\tan^{-1} n}{n} = 1$ (D) $\lim_{n \rightarrow \infty} [\ln(2n^2 + 1) - \ln(n^2 + 1)] = 2$

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2. Suppose that $f'(x) > 0$ for all $x \in \mathbb{R}$ and that $f(2) = 0$. Let $g(x) = \int_0^x f(t)dt$. Which of the following must be true?

- (A) The graph of g is increasing on $(0,2)$.
- (B) g is a nonnegative continuous function.
- (C) The graph of g has a horizontal tangent at $x = 2$.
- (D) The graph of g does not have any inflection point.

3. Suppose that series $\sum_{n=0}^{\infty} a_n x^n$ converges when $x = -3$ and diverges when $x = 5$. Which of the following must be true?

- (A) $\sum_{n=0}^{\infty} a_n$ converges.
- (B) $\sum_{n=0}^{\infty} a_n (-2)^n$ converges.
- (C) $\sum_{n=0}^{\infty} a_n (-5)^n$ diverges.
- (D) $\sum_{n=0}^{\infty} a_n 6^n$ diverges.

4. Let $f(x, y) = 4 + x^3 + y^3 - 3xy$. Which of the following must be true?

- (A) f has a local maximum at $(0,0)$.
- (B) f has a local minimum at $(1,1)$.
- (C) At the point $P(1,0)$, f have the maximum rate of change in the direction $\frac{1}{\sqrt{2}}(1, -1)$.
- (D) The maximum rate of change at the point $P(1,0)$ is $\sqrt{2}$.

三、填充題(每個空格8分, 共40分)

1. Evaluate the integral $\int_{1/3}^3 \frac{\sqrt{x}}{x^2 + x} dx$. \Rightarrow (a)

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2. Find the first three nonzero terms in the Maclaurin series for the function $f(x) = e^{-x} \ln(1+x)$. \Rightarrow (b)
3. Find the area of the part of the sphere $x^2 + y^2 + z^2 = 4y$ that lies inside the paraboloid $y = x^2 + z^2$. \Rightarrow (c)
4. Let D be the region enclosed by $y = \frac{9}{x^2+9}$, $y = 0$, $x = 0$, and $x = 3$. Find the volume of the solid obtained by rotating about the x -axis. \Rightarrow (d)
5. Let $F(x, y)$ be the vector field given by $F(x, y) = \left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right)$. Calculate $\oint_C F \cdot dr$, where C is the unit circle centered at the origin and oriented counterclockwise. \Rightarrow (e)

四、計算題(20 分) 必須有計算過程, 僅有答案而沒有計算過程得 0 分

1. (10 分) Use Lagrange multipliers to find the maximum and minimum values of $f(x, y, z) = x^2 + y^2 + z^2$ subject to the constraints $y - z = 1$ and $z^2 - x^2 = 1$.
2. (10 分) Let E be the solid bounded by the paraboloid $z = 6 - x^2 - y^2$ and the surface $z = \sqrt{x^2 + y^2}$ and let S be the boundary surface of E , given with positive (outward) orientation. Sketch the solid E and use the Divergence Theorem to find $\iint_S F \cdot dS$ for the vector field $F(x, y, z) = (x^3, y^3, 3xy)$

1. (30%) Problems with short answer
 - a. What is the main difference between the ASCII and Unicode character sets?
 - b. Distinguish between machine language and assembly language.
 - c. Why is a faster clock not always better?
 - d. What is embedded system?
 - e. What is the function of the ALU?

2. (15%) Perform the following computing.
 - a. Convert 45 (decimal number) to binary.
 - b. Convert 1111 (binary number) to octal.
 - c. Convert E7 (hexadecimal number) to decimal.
 - d. $770+665$ (octal)
 - e. A9F8-1492 (hexadecimal)

3. (15%) Show the output displayed by these nested loops:

```
for (i = 0; i < 2; ++i) {  
    printf("outer % 4d\n", i)  
    for (j = 0; j < 3; ++j) {  
        printf( " Inner%3d%3d\n", i, j);  
    }  
    for (k = 3; k > 0; --k) {  
        printf(" Inner%3d%3d\n", i, k);  
    }  
}
```

4. (15%) Rewrite the following code using a do-while statement with no decision in the loop body:

```
sum=0;  
for (even = 2, even < n; even =even +2)  
    sum = sum +even;  
print("Sum of the positive even numbers less than %d is %d\n", n, sum);
```

In what situation will the rewritten code print an incorrect sum?

5. (25%) The value for π can be determined by the series equation

$$\pi = 4 \times \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \frac{1}{13} - \dots \right)$$

Write a C program to approximate the value of π using the formula given including terms up to $1/99$.