國立中正大學
114 學年度碩士班招生考試
試 題
[第 3 節]

科目名稱	通訊理論
系所組別	通訊工程學系-通訊甲組

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始 40 分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

國立中正大學114學年度碩士班招生考試試題

1. (20 %) Let $X(f) \triangleq \int_{t=-\infty}^{\infty} x(t)\, e^{-j2\pi f t} dt$ be the spectrum, i.e., the continuous time Fourier transform (CTFT), of a deterministic complex-valued continuous energy signal $x(t)$. Consider two such signals, $x(t)$ and $y(t)$, with spectra $X(f)$ and $Y(f)$, respectively.

   (a) (10%) Describe and prove the Parseval's Theorem for $x(t)$ and $X(f)$.

   (b) (10%) Describe and prove the convolution property for $x(t)$, $y(t)$, $X(f)$, and $Y(f)$.

2. (20%) Consider the deterministic energy signal $x(t) = \text{sinc}(2f_0 t)$ and power signal $y(t) = \cos(2\pi f_0 t)$, where f_0 is a fixed number.

   (a) (10%) Compute the autocorrelation function and energy spectral density of $x(t)$.

   (b) (10%) Compute the autocorrelation function and power spectral density of $y(t)$.

3. (20 %) Consider the bandpass single side band (SSB) signal

$$x_c(t) = \frac{1}{2} A_c m(t)\cos(2\pi f_c t) - \frac{1}{2} A_c \hat{m}(t)\sin(2\pi f_c t),$$

   where $m(t)$ is the message signal, $\hat{m}(t)$ is the Hilbert transform of $m(t)$, and f_c is the carrier frequency. Let $x_{LP}(t)$ be the lowpass complex envelope of $x_c(t)$.

   (a) (10%) What is the spectrum of $x_c(t)$ when $m(t) = \text{sinc}(2f_0 t),\ f_0 << f_c$?

   (b) (10%) What is the spectrum of $x_{LP}(t)$ when $m(t) = \text{sinc}(2f_0 t),\ f_0 << f_c$?

4. (20 %) A transmitter sends a <u>complex-valued</u> symbol s with $E\{|s|^2\} = E_s$ over a wireless channel. The receiver has K antennas with received signals $x_k = h_k s + n_k,\ k = 1,2,\cdots,K$. Each h_k denotes the complex-valued channel gain and n_k is the independent complex-valued zero-mean Gaussian noise with variance $E\{|n_k|^2\} = \sigma^2$ at the k-th receive antenna.

   (a) (10%) The channel gains $h_k, \forall k$, are known. The signal at the linear combiner output is

   $$z = \sum_{k=1}^{K} w_k^* x_k ,$$

   where each w_k is the combining weight and $(\ )^*$ denotes the complex conjugate operation. What is the signal-to-noise ratio (SNR) of z for a particular set of combining weights, $w_k, \forall k$?

   (b) (10%) What values of $w_k, \forall k$, lead to the maximum SNR in part (a)?

5. (20 %) Consider the transmission of <u>complex-valued</u> symbol s over the AWGN channel $x = s + w$, where s is taken equally probably from $\mathcal{S} \triangleq \{1+j,\ 1-j,\ -1+j,\ -1-j\}$ and w is circularly Gaussian distributed with zero mean and variance N_0.

   (a) (3 %) What is the likelihood function of receiving x when a particular value of s from $\mathcal{S}$ is transmitted?

   (b) (7 %) What is maximum likelihood detection (MLD) of s from observed x?

   (c) (10 %) What is the probability $P\{\hat{x} \neq 1+j \mid s = 1+j\}$, i.e. the probability that $s = 1+j$ is transmitted but $\hat{x} \neq 1+j$ is detected, for the MLD in part (b)?

國立中正大學
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試 題
[第 4 節]

科目名稱	線性代數與機率
系所組別	通訊工程學系-通訊甲組

—作答注意事項—

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線性代數試題

1. (10%) A matrix B is said to be a square root of a matrix A if $BB=A$. (1) Find square roots of $A = \begin{bmatrix} 5 & 5 \\ 5 & 10 \end{bmatrix}$. (2) Prove that for a 2×2 matrix A whose determinant is negative, then A has no real square root.

2. (10%) Show that the following matrices form a basis for M_{22}.

$$\begin{bmatrix} 3 & 4 \\ 3 & -4 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -8 \\ -12 & -2 \end{bmatrix}$$

3. (10%) Are there values of r and s for which

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & r-2 & 2 \\ 0 & s-1 & r+2 \\ 0 & 0 & 3 \end{bmatrix}$$

has rank 1? Has rank 2? Has rank 3? Has rank 4?

4. (10%) Find a matrix S such that $S^2 = A$, given that $A = \begin{bmatrix} 1 & 3 & 5 \\ 0 & 4 & 5 \\ 0 & 0 & 9 \end{bmatrix}$

5. (10%) In R^3, consider the line l given by the equations

$$x = -1, y = t, z = -t$$

and the line m given the equation

$$x = 2s, \quad y = 1 + s, z = s$$

Let P be a point on l and Q be a point on m, find the values of t and s such that $\|P - Q\|^2$ is minimized.

機率試題

6. (32%) The waiting time X of a customer at a taxi stand is zero if the customer finds a taxi parked at the stand, and a uniformly distributed random length of time in the interval $[0,1]$ (in hours) if no taxi is found upon arrival. The probability that a taxi is at the stand when the customer arrives is equal to 0.7. Let the cumulative distribution function (cdf) of X be represented in the form:

$$F_X(x) = pF_Y(x) + (1-p)F_Z(x),$$

where $0 < p < 1$ and $F_Y(x)$ is the cdf of *continuous* random variable Y, while $F_Z(x)$ is the cdf of *discrete* random variable Z. Find the following:

 a) (4%) $F_Y(x)$.
 b) (4%) p.
 c) (4%) $F_Z(x)$.
 d) (4%) Probability density function (pdf) of Y.
 e) (4%) Probability mass function of Z.
 f) (6%) Mean of X.
 g) (6%) Variance of X.

7. (18%) Let X be a continuous random variable with cdf $F_X(x)$ and pdf $f_X(x)$. Consider $Y = \sqrt{X}$.

 a) (4%) The event $\{Y \leq y\}$ is equivalent to what event involving X itself?
 b) (4%) Use part a) to find the cdf of Y in terms of $F_X(x)$.
 c) (4%) Use part b) to find the pdf of Y in terms of $f_X(x)$.
 d) (6%) If X denotes the squared envelope of a radio signal with the following pdf:

$$f_X(x) = \frac{1}{2\alpha^2}e^{-x/2\alpha^2},\ x > 0,\ \alpha > 0,$$

 where α is a constant, use part c) to find the pdf of the envelope Y.