國立中正大學
111 學年度碩士班招生考試
試 題
[第 1 節]

科目名稱	通訊原理
系所組別	通訊工程學系-通訊甲組

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

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國立中正大學111學年度碩士班招生考試試題

科目名稱：通訊原理　　　　　　　　　　　　　　　　　　本科目共 2 頁　第 1 頁

系所組別：通訊工程學系-通訊甲組

1. (15 %) A quadrature-carrier multiplexing is employed to transmit two message signals $m_1(t)$ and $m_2(t)$ over the same carrier frequency f_c. The transmitted signal is given by

$$u(t) = A_c m_1(t)\cos(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t).$$

Sketch the demodulator and explain how to demodulate the message signals $m_1(t)$ and $m_2(t)$ from $u(t)$.

2. (20 %) Let $W(t)$ be a white noise with power spectral density $N_0/2$. Assume that $W(t)$ is passed through an ideal bandpass filter with frequency response $H(f)$ given by

$$H(f) = \begin{cases} 1, & |f \pm f_c| < B \\ 0, & \text{otherwise} \end{cases}$$

Let $N(t)$ be the output random process of the filter.

A. (5 %) Determine $E[|N(t)|^2]$.

B. (5 %) The random process $N(t)$ can be represented by

$$N(t) = N_I(t)\cos(2\pi f_c t) - N_Q(t)\sin(2\pi f_c t).$$

Find the power spectral density of $N_I(t)$ and $N_Q(t)$.

C. (5 %) Let $Z(t) = \frac{d}{dt}N_I(t)$. Determine the power spectral density of $Z(t)$.

D. (5 %) Determine $E[|Z(t)|^2]$.

3. (20 %) The M signal waveforms for a M-ary FSK may be expressed as

$$s_m(t) = \sqrt{\frac{2E_s}{T}}\cos(2\pi f_c t + 2\pi m\Delta f t),$$

for $m = 0,1,\ldots,M-1$ and $0 \le t \le T$, where f_c is the carrier frequency and Δf is the frequency separation between successive frequencies. Assume that $f_c = n/T$ with n being an integer.

A. (5%) Determine the correlation coefficients

$$\gamma_{mn} = \frac{1}{E_s}\int_0^T s_m(t)s_n(t)dt$$

B. (5%) Determine the minimum frequency separation between successive frequencies for orthogonality.

C. (5%) Sketch the correlator-type receiver.

D. (5%) At the receiver, assume that the received signal is corrupted by AWGN. Derive the ML decision rule for the M-ary FSK system.

4. (25 %) Consider the $M = 4$ biorthogonal signals shown in Figure 1 for transmitting information over an AWGN channel. The noise is assumed to have zero mean and power spectral density $N_0/2$.

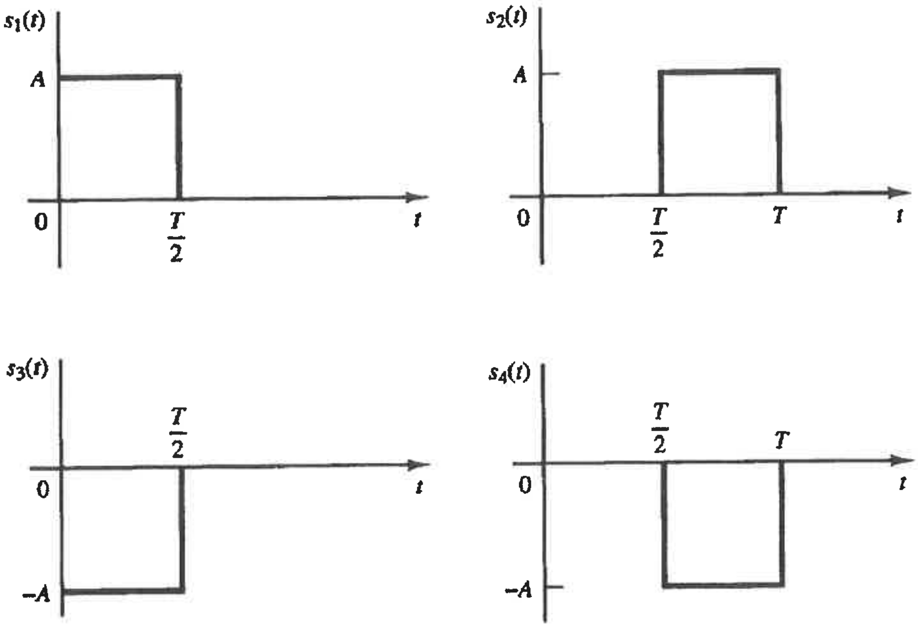

Figure 1

A. (5 %) Determine and sketch the basis functions for this signal set.

B. (5 %) Sketch the impulse responses of the matched-filters that match to the basis functions.

C. (5 %) If the noise variance is zero, sketch the output waveforms of the matched-filters when the transmit signal is $s_1(t)$.

D. (10 %) Determine the symbol error rate of the system if an ML detector is employed.

5. (20 %) Let $\{b_n\}$ be uncorrelated binary valued $\{+1,-1\}$ random process with $P\{b_n = +1\} = P\{b_n = -1\} = 1/2$. From $\{b_n\}$, we form the random process

$$a_n = b_n - b_{n-1}.$$

The transmit signal is given by

$$v(t) = \sum_{n=-\infty}^{\infty} a_n g(t - nT),$$

where $g(t)$ is the impulse response of the transmitting filter (a.k.a. pulse shaping function) given by

$$g(t) = \begin{cases} 1, & 0 \le t < T, \\ 0, & \text{otherwise.} \end{cases}$$

A. (5 %) Determine the autocorrelation function of $\{a_n\}$.

B. (15 %) Determine the power-spectral density of $v(t)$.

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[第 2 節]

科目名稱	機率
系所組別	通訊工程學系-通訊甲組

—作答注意事項—

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科目名稱：機率　　　　　　　　　　　　　　　　　　　　　　　　本科目共 1 頁　第 1 頁
系所組別：通訊工程學系-通訊甲組

1) **(10%)** A number x is selected at random in the interval $[-1, 2]$. Numbers from the subinterval $[-1, 0)$ occur half as frequently as those from $[0, 2]$.
   a) **(5%)** Find the probability that x is less than zero.
   b) **(5%)** Find the probability that x is less than zero, given that the absolute value of x is less than 1.

2) **(20%)** Let N be a geometric random variable taking on values in $\{1, 2, \ldots\}$.
   a) **(5%)** Find $P[N = k]$.
   b) **(5%)** Find $P[N > k]$.
   c) **(5%)** Find the mean of N.
   d) **(5%)** Find the probability that N is even.

3) **(10%)** Let X be a discrete random variable with the following probability mass function:

$$p_X(k) = e^{-\pi}\frac{\pi^k}{k!} \quad \text{for all} \quad k = 0, 1, 2, \ldots.$$

   a) **(5%)** Find the mean of X.
   b) **(5%)** Find the variance of X.

4) **(15%)** A random variable X has a continuous cumulative distribution function (cdf) $F_X(x)$:

$$F_X(x) = \begin{cases} 0, & \text{for } x < 0, \\ 1 - \frac{1}{2}e^{-2x}, & \text{for } x \geq 0. \end{cases}$$

   a) **(5%)** Identify the type of random variable.
   b) **(5%)** Find $P[X \leq 0]$.
   c) **(5%)** Find $F_X(x \mid C)$ where $C = \{X > 0\}$.

5) **(10%)** U is selected uniformly at random from the unit interval $[0, 1]$; X is then selected uniformly at random from the interval $(0, U)$.
   a) **(5%)** Find $P[X \leq x \mid U = u]$.
   b) **(5%)** Find the cdf of X.

6) **(10%)** Let $X = U^n$ where n is a positive integer and U is a uniform random variable in the unit interval $[0, 1]$.
   a) **(5%)** Find the cdf of X.
   b) **(5%)** Find the probability density function (pdf) of X.

7) **(25%)** Let X and Y be jointly Gaussian random variables with the following pdf:

$$f_{X,Y}(x, y) = \frac{\exp\{-2x^2 - 9y^2/2 - 9y - 9/2\}}{2\pi c} \quad \text{for all} \quad x, y.$$

   a) **(5%)** Find the constant c.
   b) **(5%)** Find the mean of Y.
   c) **(5%)** Find the variance of X.
   d) **(5%)** Find the covariance of X and Y.
   e) **(5%)** Find the marginal pdf for X.

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科目名稱	線性代數
系所組別	通訊工程學系-通訊甲組

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1. Let $\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & 2 & 0 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$. Find the results with details.

a. (5 pts.) The determinant.

b. (10 pts.) The inverse matrix.

c. (10 pts.) The eigenvalues and their corresponding eigenvectors.

d. (10 pts.) Find a matrix $\boldsymbol{P}$ that diagonalizes $\boldsymbol{A}$.

e. (5 pts.) Find $\boldsymbol{A}^7$.

f. (20 pts.) Find QR-decomposition of $\boldsymbol{A}$.

g. (10 pts.) Are the row vectors of $\boldsymbol{A}$ linearly dependent? Explain your answers.

h. (5 pts.) Find the orthogonal complement of the null space of $\boldsymbol{A}$.

2. (25 pts.) Prove that a singular matrix with size $n\times n$ cannot be reduced as $\boldsymbol{I}_n$ by Gauss-Jordan elimination.
(Hint: You can use the skill of contradiction to prove this rule)
If you have no idea about this proof, please show a 3x3 matrix without zero row and column as an example to explain this rule. (Just get some points)